

Differentiable Dynamics for Autonomous Micro-Mobility Navigation

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Motivation

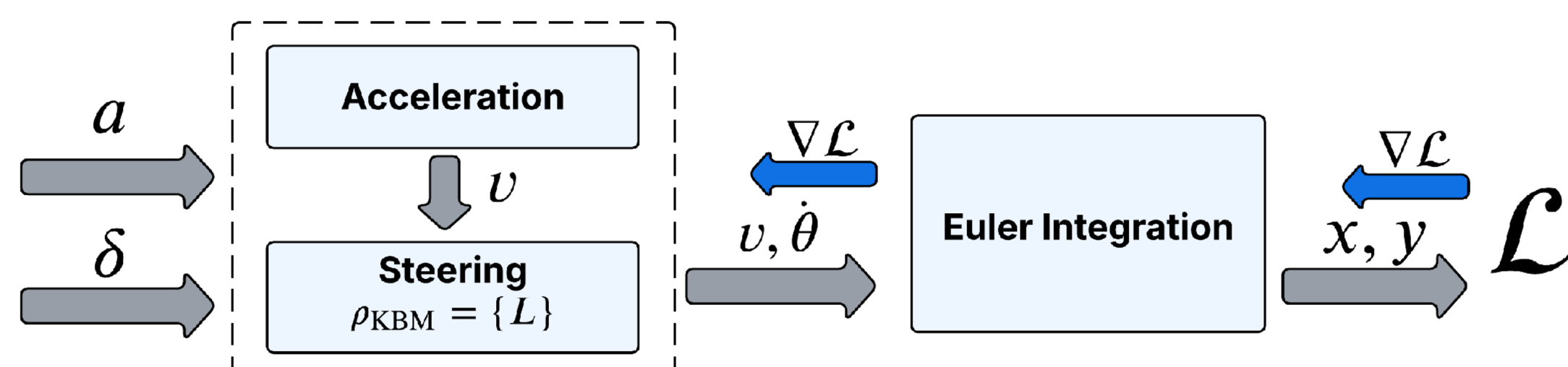
- Autonomous micro-mobility vehicles (MMVs) — wheelchairs, scooters, bicycles — must move predictably in pedestrian-shared spaces. Poorly modeled dynamics cause over-/under-steering and abrupt stops.
- Differentiable AV stacks such as DiffStack rely on unicycle or kinematic bicycle models: **no tire slip, friction, load transfer, or lean**.
- The **General Micro-Mobility Model (GM3)** models tire-level MMV dynamics for arbitrary wheel layouts, but is **non-differentiable** so it cannot be trained inside a gradient-based stack.

Contributions

- DiffKBM**: the classical kinematic bicycle model (KBM), fully differentiable, with learnable wheelbase L .
- DiffGM3**: a differentiable reformulation of GM3 with smoothed brush-tire forces, singularity-free steering, and lean as a second-order DOF.
- Framework for learning physical parameters that are hard to measure: friction, contact length, stiffness, inertias, damping.
- Dynamics layer for open-loop imitation in DiffStack and closed-loop planning with differentiable MPC (DiffMPC).

DiffKBM Baseline

We reformulate the already differentiable KBM so wheelbase L is learnable.



Making GM3 Differentiable

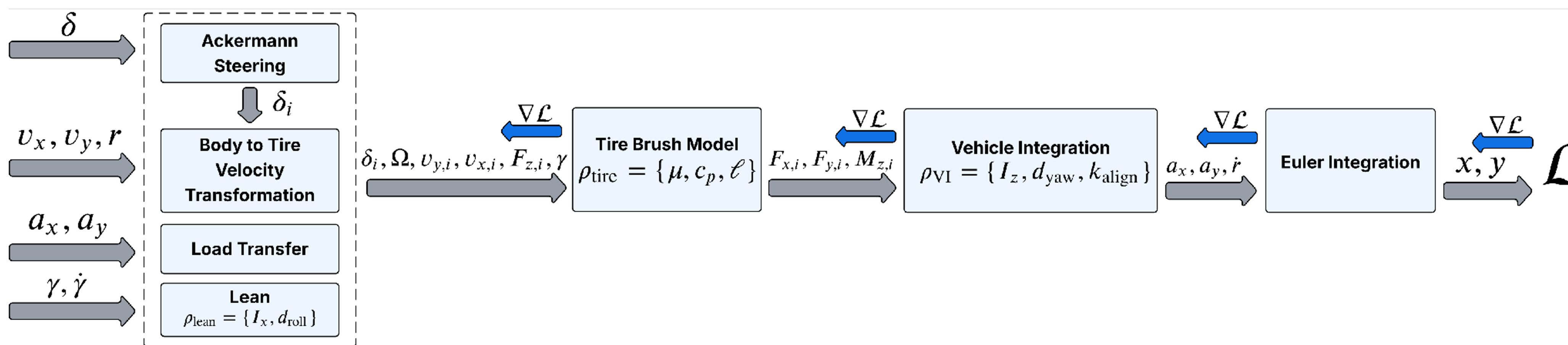
GM3's brush-model forces switch at the adhesion-sliding boundary. Every discontinuity is replaced by a smooth surrogate so gradients flow.

Discontinuity	Smooth surrogate
Adhesion / sliding switch at $\sigma = \frac{1}{\psi}$	sigmoid gate $G = \frac{1}{1 + e^{-k(\sigma - 1/\psi)}}$
$\text{sgn}(\alpha)$ in F_y and ψ^*	$\tanh(k\alpha)$
$ \sigma $ at zero slip	$\sqrt{\sigma_x^2 + \sigma_y^2 + \epsilon}$
Ackermann singularity at $\delta = 0$	$\delta_i = \arctan\left(\frac{L \tan \delta}{L - y_i \tan \delta}\right)$
Hard parameter / control limits	$\tanh$ soft clamp



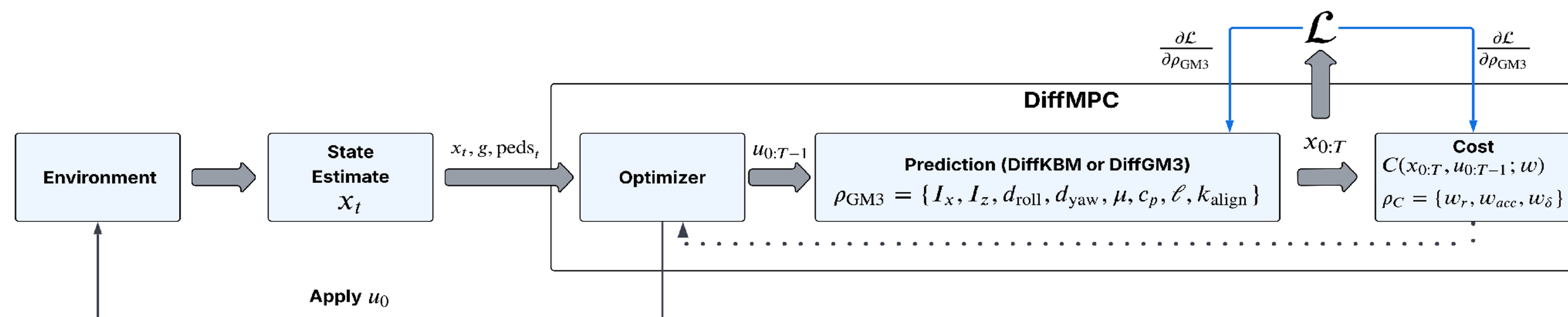
Website: gm3-sim.github.io/diffgm3
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DiffGM3 Architecture



Fully differentiable pipeline: the steering input δ is mapped to per-tire angles; tire velocities, load transfer, and lean feed the smoothed brush model; per-tire forces integrate to vehicle acceleration and an Euler-integrated pose. Gradients reach every module, so parameter sets ρ_{lean} , ρ_{tire} , and ρ_{v1} train directly on MMV trajectory data.

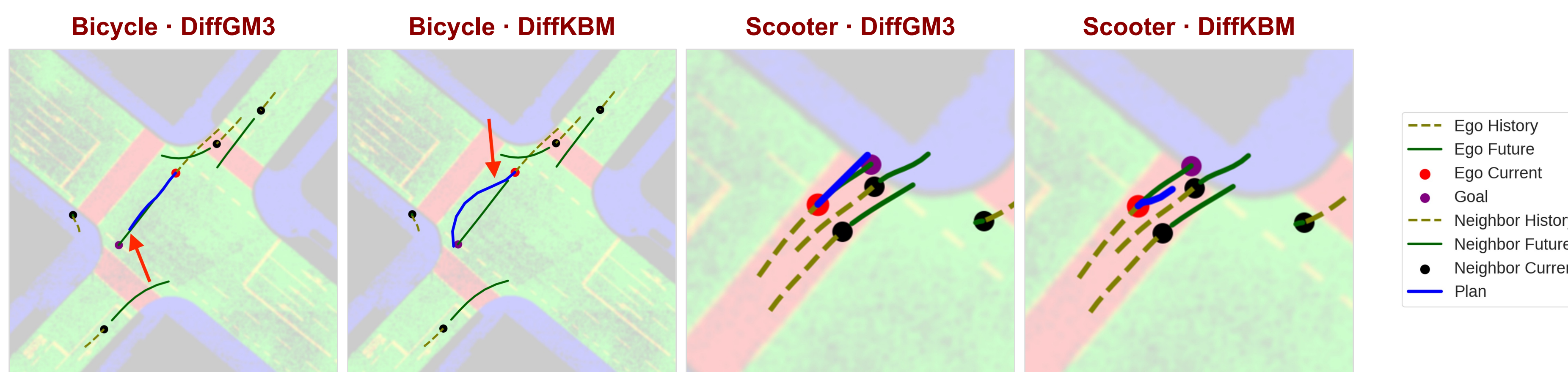
DiffMPC + DiffGM3



We pair DiffGM3 with a differentiable MPC controller (**DiffMPC**) and evaluate it in **CrowdNav**, a crowd-robot interaction benchmark where an ego vehicle must reach a goal while avoiding multiple moving pedestrians. At each timestep, the optimizer rolls candidate controls through DiffKBM or DiffGM3, scores them with a differentiable cost (goal tracking, turning-circle CBF, control regularization), and applies u_0 in a receding-horizon loop. Gradients flow through both the dynamics and the cost.

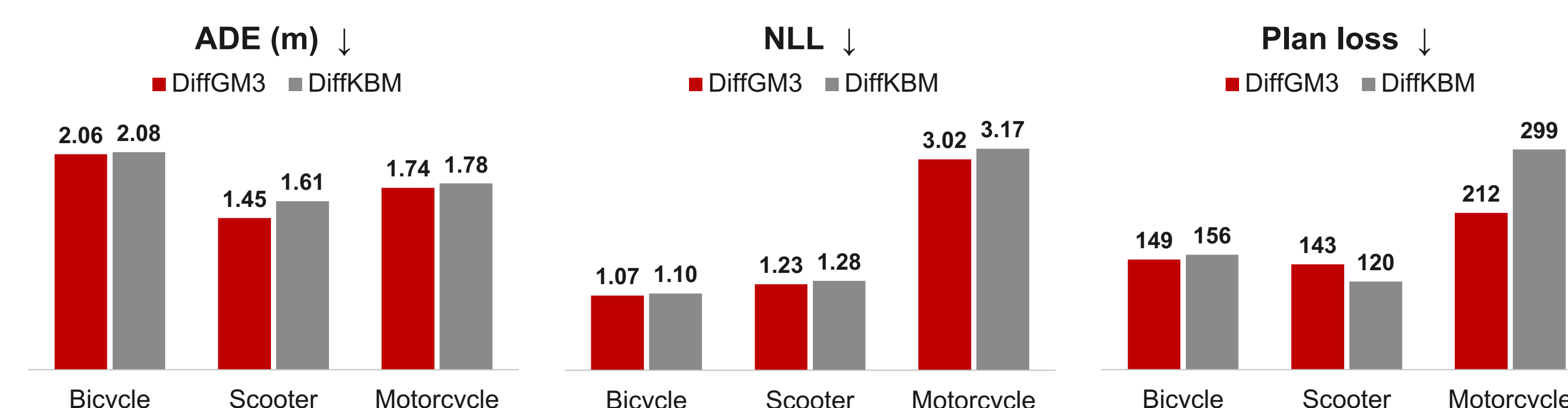
Open-loop Trajectory Matching

DiffKBM and DiffGM3 replace the dynamically-extended unicycle model inside DiffStack and are trained to reproduce real intersection trajectories from the IMPTC dataset, including 617 bicycle, 233 motorcycle, and 85 scooter tracks.



DiffGM3 stays on the ground-truth path because damping and align gains prevent overturning; DiffKBM reaches the goal but veers off-path (bicycle) or turns into another agent and stalls (scooter).

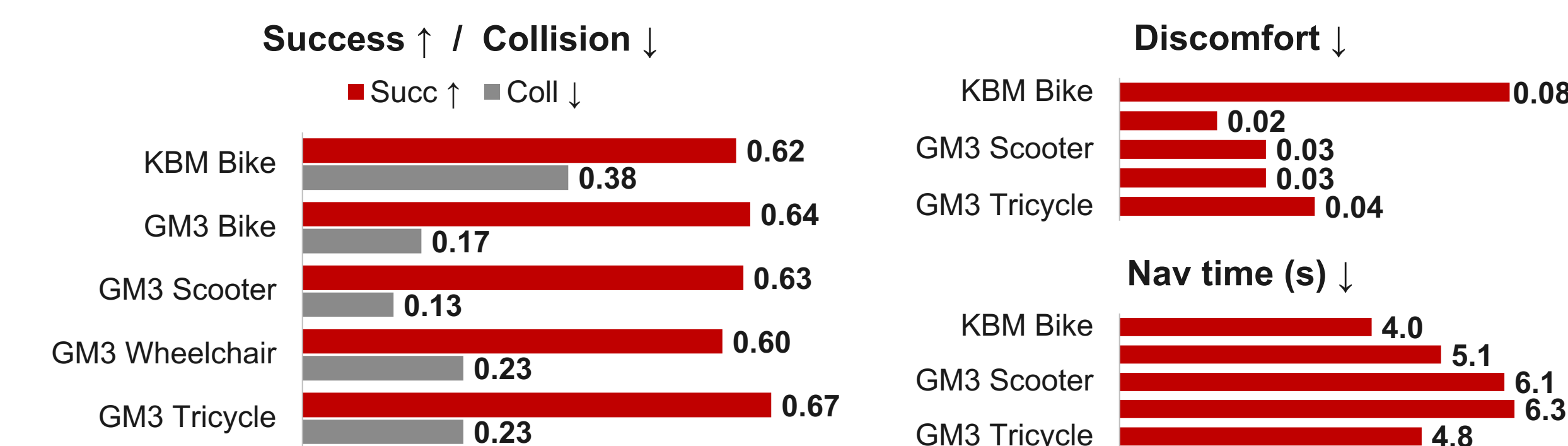
Open-loop Results



DiffGM3 outperforms DiffKBM on ADE and NLL for every mode. The scooter planning-loss regression comes from DiffGM3 braking before the goal — it holds the path while DiffKBM swerves off and turns back.

Closed-loop Results

DiffMPC with DiffGM3 and DiffKBM was evaluated on 100 CrowdNav pedestrian scenarios for bicycle, scooter, wheelchair, and tricycle. DiffGM3's longer navigation times reflect veering around pedestrians; DiffKBM's faster times are due to collisions ending runs early.

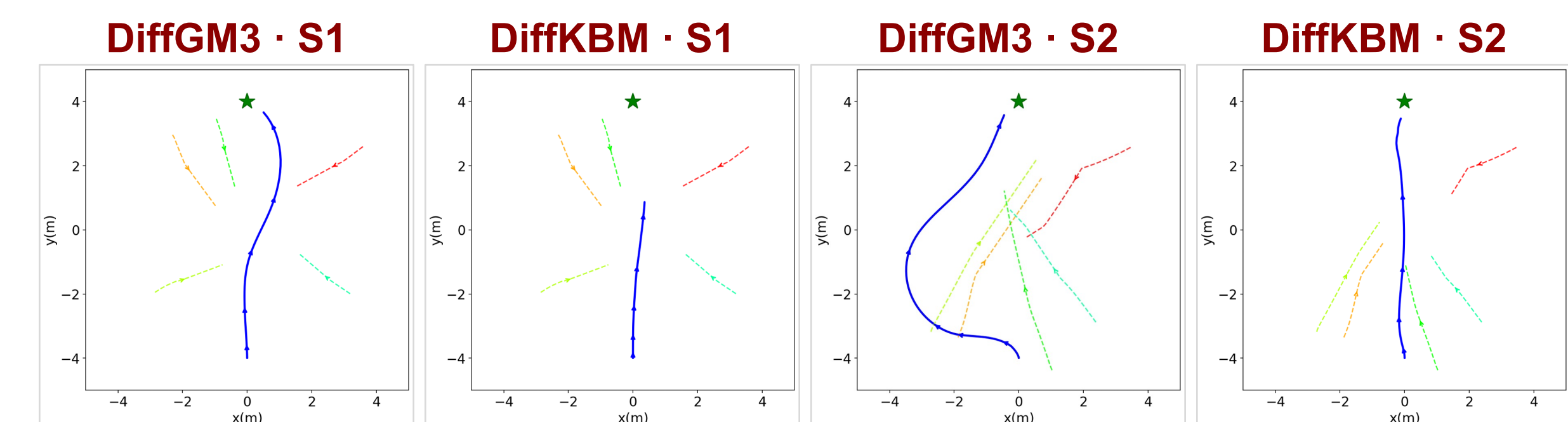


DiffGM3 vs. DiffKBM bicycle: 55% fewer collisions, 75% lower discomfort frequency, +3% success rate.

Closed-loop Scenarios



Closed-loop Scenario 1: DiffKBM approaches the pedestrian group (left) while DiffGM3 steers around it (right).



Scenario 1: DiffKBM squeezes between two pedestrians and collides; DiffGM3 brakes and steers around them.
Scenario 2: both reach the goal, but DiffGM3's longer curve holds more distance and lower discomfort.

Conclusion

- A unified tire-level MMV model can be made fully differentiable and implemented in a controller, allowing gradients to flow through physical parameters and cost functions.
- DiffGM3 reproduces real trajectories more accurately than DiffKBM and navigates crowds far more safely using one formulation shared by bicycles, scooters, tricycles, and wheelchairs.
- Future work: high-curvature and long-horizon data, differentiable simulation with multiple MMV types, and hardware deployment.

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